Axiomatic Design of Automotive Suspension Systems

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Abstract
This paper presents kinematic design methodology of a suspension system using Axiomatic Design (AD). AD is applied to typical three types of the front suspension systems: McPherson strut, double wishbone and multilink. Our study includes the analysis of the functional independencies of current suspension design configurations, which would add to the understanding of how various suspension hardpoints influence the suspension functional requirements (FRs). In addition, this paper also proposes sequential design orders in suspension kinematic design to satisfy all of the suspension FRs. Of the current kinematic designs, the multilink is a decoupled design, whereas McPherson strut and double wishbone are coupled designs. It is shown that a coupled design can be decoupled by applying the independence axiom. The design matrices formulated for the suspension systems indicate a specific design order to satisfy all FRs.

Keywords: Axiomatic Design, Kinematics, Suspension System

1 INTRODUCTION

Automotive suspension is a collection of rigid bodies moving relative to each another. It is a typical example of a coupled system, designed by the experience of expert designers and by trial and error. Although various suspension configurations have been developed to improve vehicle performances, only a fraction of all technical research results of these configurations have been reported. The kinematic design of a suspension system begins with determining the suspension layout [1]. With no established design methodologies, designers have always relied on their know-how to make improvements. For this reason, this paper discusses the well-designed suspension systems and how it can be further improved using the Axiomatic Design (AD) approach.

The AD framework [2], which consists of two axioms, provides the fundamental axioms for analysis and decision-making and introduces a systematic approach to the design process, which has usually employed empirical and *ad hoc* methods. These two axioms are very effective in the conceptual design stage as well as in the detailed design stage. These axioms embody two essential concepts: the independence of functional requirements (the independent axiom) and the minimization of information content (the information axiom). AD can be applied to various design problems and has extended to a wide range of engineering designs, such as product design [2], manufacturing system [3,4], system design [5], software design [6,7] and control system design [8].

In this paper, the existing suspension designs are compared using the AD theory to classify good design from the bad ones in the conceptual design stage. Moreover, the AD theory is applied to the detailed kinematic design of a suspension system. Sequential design orders will be determined from the design matrices of several suspension types and compared with each other.

2 DESIGN EQUATIONS OF SUSPENSION SYSTEMS

In this section, kinematic functional requirements (FRs) of suspension systems will be defined and the corresponding design equations will be derived. The kinematic characteristic of a suspension system is called suspension geometry, which is related with the motion of a wheel assembly when it moves up and down. The path of relative wheel motion is governed by the layout of the suspension links. Moreover, the steering layout plays an important role in front suspension systems. Among the various suspension geometries, three independent geometries are considered, and the front and side views of the kingpin axis are also considered for steering layout. Hence, the kinematic FRs of the suspension design may be stated as follows:

- FR_1 = maintain desired toe angle change.
- FR_2 = maintain desired camber angle change.
- FR_3 = maintain desired wheelbase change.
- FR_4 = maintain desired kingpin offset.
- FR_5 = maintain desired caster angle.

The design parameters (DPs) are generally selected as follows:

- DP_1 = toe control link (tie rod).
- DP_2 = front view of suspension layout.
- DP_3 = side view of suspension layout.
- DP_4 = front view of kingpin axis.
- DP_5 = side view of kingpin axis.

The above-mentioned DPs are theoretically sound, but physically meaningless. Moreover, it is difficult to select such DPs in the suspension design stage. In the kinematic design of a suspension system, the coordinates of the hardpoints may be considered as DPs. Difficulties arise from the fact that the designer cannot expect which hardpoints will affect the specific FRs. This paper will discuss the selection process of DPs from hardpoints, and the AD will be employed to define the relationship between hardpoints and suspension geometries.

Design equation of a suspension system can be derived from the kinematic equation, which is written as a set of nonlinear algebraic equations [9].

$$\mathbf{\Phi}(\mathbf{q}) = \mathbf{0} \tag{1}$$

where $\mathbf{\Phi}$ is a set of constraint equations and $\mathbf{q}$ is a set of generalized coordinates. Suspension geometries can be calculated from Eq. (1). To mathematically express the FRs, we consider the performance index such that

$$\mathrm{FR}_i \equiv \frac{\int (C_i - \overline{C}_i)^2 dz}{\int \overline{C}_i^2 dz} \tag{2}$$

where C_i stands for each curve of the kinematic objectives in design state, and $\overline{C}_i$ stands for the desired curve.

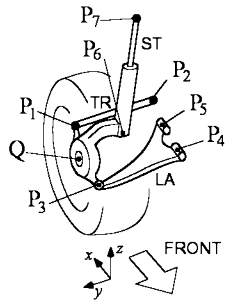
Design equation of a suspension system can be derived from Eqs. (1) and (2). Since the kinematic equation of Eq. (1) is highly nonlinear, the design matrix may be written in a differential form such as

$$\begin{Bmatrix} \Delta\mathrm{FR}_1 \\ \vdots \\ \Delta\mathrm{FR}_m \end{Bmatrix} = \begin{bmatrix} \mathrm{A}_{11} & \cdots & \mathrm{A}_{1n} \\ \vdots & \ddots & \vdots \\ \mathrm{A}_{m1} & \cdots & \mathrm{A}_{mn} \end{bmatrix} \begin{Bmatrix} \Delta\mathrm{DP}_1 \\ \vdots \\ \Delta\mathrm{DP}_n \end{Bmatrix} \tag{3}$$

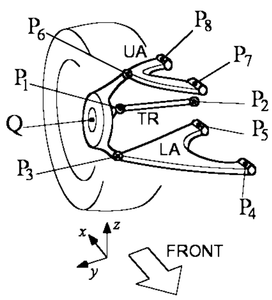
where $\mathrm{A}_{ij} = \partial\mathrm{FR}_i / \partial\mathrm{DP}_j$. In the kinematic design of a suspension system, the number of hardpoints is generally more than that of FRs. In this case, the design matrix is written in row-wise rectangular matrix which is in the form of $m < n$. When there are more DPs than FRs, the design is a redundant design (Theorem 3 in [2]). According to Theorem 4 in [2], an ideal design can be achieved by two possible methods: *grouping* of the effective DPs and *freezing* of the unnecessary DPs. In this paper, the two methods are combined so that the dominant DPs are selected and then grouped to form new DPs. When grouping the DPs, we should select new DPs to maximize the quantitative measures [10].

3 CASE STUDIES

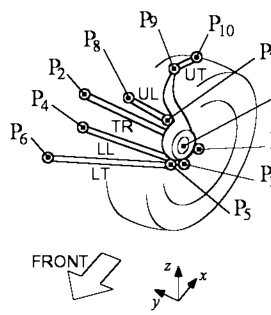
The AD is applied to the three typical types of suspension systems, the McPherson strut, double wishbone and multilink as shown in Figure 1. These applications are briefly discussed here, and detailed descriptions can be found in reference [9].



(a) McPherson strut



(b) Double wishbone



(c) Multilink

Figure 1: Front suspension systems.

3.1 McPherson strut

The McPherson strut suspension is the most popular front suspension system. This type of suspension consists of an arm (LA), a strut (ST) and a link (TR) as shown in Figure 1(a). It has 7 hardpoints (P_1 ~ P_7) which have three coordinates respectively, resulting 21 DPs. The design matrix, which is calculated from the sensitivity of the initial design state, is shown in Table 1. In Table 1, only the effective DPs are shown for simplicity, and each element is symbolized as either X (strong), ⊙ (passable) or 0 (weak). Positive and negative signs in Table 1 represent the sign of each sensitivity value. Checked (✓) ones are selected as the dominant DPs for each FR.

Table 1: Effect of DPs on FRs for the McPherson strut.

H/P	DPs	FR_1	FR_2	FR_3	FR_4	FR_5
P_1	DP_3	-X ✓	+0	+⊙	+0	-0
P_2	DP_6	+X ✓	-0	-⊙	-0	+0
	DP_7	+0	+0	-0	-0	+X ✓
P_3	DP_8	+0	+0	+0	+X ✓	+0
	DP_9	+X	+X	-⊙	+⊙ ✓	-0
P_4	DP_{12}	-X	-X	+X ✓	+0	+0
P_5	DP_{15}	-0	+0	-X ✓	-0	-0
P_6	DP_{17}	+⊙	-X ✓	-0	-0	+0
P_7	DP_{19}	+0	-0	+0	+0	-X ✓

To decouple the design matrix, the effective DPs are separated from the negligible DPs. Since the number of selected hardpoints is still greater than that of FRs, new DPs are grouped as follows:

$DP'_1 = DP_3$ and DP_6

$DP'_2 = DP_{17}$

$DP'_3 = DP_{12}$ and DP_{15}

$DP'_4 = DP_8$ and DP_9

$DP'_5 = DP_7$ and DP_{19}

After grouping and rearranging, the design equation for the new set of DPs may be written as:

$$\begin{Bmatrix} \mathrm{FR}_5 \\ \mathrm{FR}_4 \\ \mathrm{FR}_3 \\ \mathrm{FR}_2 \\ \mathrm{FR}_1 \end{Bmatrix} = \begin{bmatrix} \times & 0 & 0 & 0 & 0 \\ 0 & \times & 0 & 0 & 0 \\ 0 & 0 & \times & 0 & \otimes \\ 0 & \times & \times & \times & 0 \\ 0 & \times & \times & 0 & \times \end{bmatrix} \begin{Bmatrix} \mathrm{DP}'_5 \\ \mathrm{DP}'_4 \\ \mathrm{DP}'_3 \\ \mathrm{DP}'_2 \\ \mathrm{DP}'_1 \end{Bmatrix} \tag{4}$$

According to Eq. (4), the McPherson strut suspension is an almost decoupled design. However, FR_1 and FR_3 are affected by two DPs (DP'_1 and DP'_3) simultaneously. This type of system is defined as a *cross-coupled* system, which cannot be decoupled at the current design.

3.2 Double wishbone

The double wishbone suspension consists of two arms (UA, LA) and a link (TR) as shown in Figure 1(b). With 8 hardpoints (P_1 ~ P_8), there are a total of 24 DPs. The design matrix is shown in Table 2.

Table 2: Effect of DPs on FRs for the double wishbone.

H/P	DPs	FR_1	FR_2	FR_3	FR_4	FR_5
P_1	DP_3	-X ✓	-0	-X	-0	-0
P_2	DP_6	+X ✓	+0	+X	+0	+0
P_3	DP_7	+0	+⊙	+0	+0	+X ✓
	DP_8	-0	-0	-0	-X ✓	-0
P_4	DP_{12}	-⊙	-⊙	-X ✓	-0	+0
P_5	DP_{15}	-⊙	-⊙	+X ✓	-0	-0
	DP_{16}	-0	+0	+0	-0	-X ✓
P_6	DP_{17}	-0	+0	-0	+X ✓	+0
	DP_{18}	+⊙	-X ✓	+0	+⊙	-0
P_7	DP_{21}	-0	+⊙ ✓	-0	-0	-0

New DPs for the double wishbone suspension type are grouped as follows:

$DP'_1 = DP_3$ and DP_6

$DP'_2 = DP_{18}$ and DP_{21}

$DP'_3 = DP_{12}$ and DP_{15}

$DP'_4 = DP_8$ and DP_{17}

$DP'_5 = DP_7$ and DP_{16}

After grouping and rearranging, the design equation for the new set of DPs may be written as:

$$\begin{Bmatrix} FR_5 \\ FR_4 \\ FR_3 \\ FR_2 \\ FR_1 \end{Bmatrix} = \begin{bmatrix} \times & 0 & 0 & 0 & 0 \\ 0 & \times & 0 & 0 & 0 \\ 0 & 0 & \times & 0 & \otimes \\ 0 & 0 & \times & \times & 0 \\ 0 & 0 & \times & \times & \times \end{bmatrix} \begin{Bmatrix} DP'_5 \\ DP'_4 \\ DP'_3 \\ DP'_2 \\ DP'_1 \end{Bmatrix} \tag{5}$$

From Eq. (5), the double wishbone suspension is also a *cross-coupled* system.

3.3 Multilink

The multilink suspension, which is shown in Figure 1(c), consists of five links (UL, UT, LL, LT, TR). With 10 hardpoints (P_1 ~ P_{10}), there are 30 DPs. It has more DPs than the other two types, which means that it has great deal of design freedom. The design matrix is shown in Table 3. In this table, P_4, P_9 and P_{10} are not selected as the dominant DPs, and therefore they are removed for limited space.

Table 3: Effect of DPs on FRs for the multilink.

H/P	DPs	FR_1	FR_2	FR_3	FR_4	FR_5
P_1	DP_3	+X ✓	+0	+⊙	+0	-0
P_2	DP_6	-X ✓	-0	-⊙	-0	+0
P_3	DP_7	-0	+0	+0	+X	+X ✓
P_5	DP_{13}	-0	+0	-0	-X ✓	+0
	DP_{14}	+0	-0	-0	+X ✓	-0
	DP_{15}	-0	+0	-X ✓	+0	-0
P_6	DP_{18}	+0	-0	+X ✓	+0	-0
P_7	DP_{19}	-0	-0	-0	+0	-X ✓
	DP_{21}	-0	-X ✓	-0	-0	+0
P_8	DP_{24}	+0	+X ✓	+0	-0	+0

New DPs are grouped as follows:

$DP'_1 = DP_3$ and DP_6

$DP'_2 = DP_{21}$ and DP_{24}

$DP'_3 = DP_{15}$ and DP_{18}

$DP'_4 = DP_{13}$ and DP_{14}

$DP'_5 = DP_7$ and DP_{19}

After grouping and rearranging, the design equation for the new set of DPs may be written as:

$$\begin{Bmatrix} FR_5 \\ FR_4 \\ FR_2 \\ FR_1 \\ FR_3 \end{Bmatrix} = \begin{bmatrix} \times & 0 & 0 & 0 & 0 \\ \times & \times & 0 & 0 & 0 \\ 0 & 0 & \times & 0 & 0 \\ 0 & 0 & 0 & \times & 0 \\ 0 & 0 & \times & \times & \times \end{bmatrix} \begin{Bmatrix} DP'_5 \\ DP'_4 \\ DP'_2 \\ DP'_1 \\ DP'_3 \end{Bmatrix} \tag{6}$$

According to Eq. (6), the multilink suspension is fully decoupled. That is, it is a good design compared with the other two types.

4 DESIGN MODIFICATION AND DECOUPLING

As stated in the previous section, the McPherson strut suspension is an almost decoupled design. Since it is a cross-coupled design, it cannot be decoupled at the current design. The basic problem arises from the fact that the wheel side hardpoint of the LA (P_3 in Figure 1(a)) affects all of the FRs. For this reason, the LA is replaced with two equivalent links (LF, LR), as shown in Figure 2, to convert it to a decoupled design.

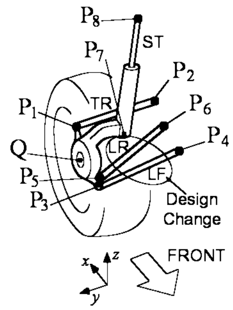


Figure 2: Design change for the McPherson strut.

The design equation for the new design of the McPherson strut may be written as:

$$\begin{Bmatrix} FR_5 \\ FR_4 \\ FR_2 \\ FR_1 \\ FR_3 \end{Bmatrix} = \begin{bmatrix} \times & 0 & 0 & 0 & 0 \\ \times & \times & 0 & 0 & 0 \\ 0 & 0 & \times & 0 & 0 \\ 0 & 0 & 0 & \times & 0 \\ 0 & 0 & 0 & \times & \times \end{bmatrix} \begin{Bmatrix} DP'_5 \\ DP'_4 \\ DP'_2 \\ DP'_1 \\ DP'_3 \end{Bmatrix} \tag{7}$$

The new design of the McPherson strut is a decoupled design. This means that the McPherson strut suspension can be converted from a coupled to a decoupled system by replacing the arm with two equivalent links.

5 COMPARISON OF FUNCTIONAL INDEPENDENCIES OF SUSPENSION SYSTEMS

The design equations of the suspension designs reveal a similarity among the designs. To compare the functional independencies, reangularity (R) and semangularity (S) [10] are computed. Table 4 shows that the multilink suspension is the best decoupled design. It can be also observed from the design equations (Eqs. (4) and (5)) and Table 4 that the McPherson strut and double wishbone show similar functional characteristics. This is because the kinematic element, a strut, merely produces the motion equivalent to an arm of infinite radius [1].

Table 4: Quantitative measures.

suspension type	R	S	R/S
McPherson strut	0.4858	0.7060	0.6881
double wishbone	0.3665	0.6354	0.5768
multilink	0.7327	0.8149	0.8991
design change	0.6772	0.8029	0.8434

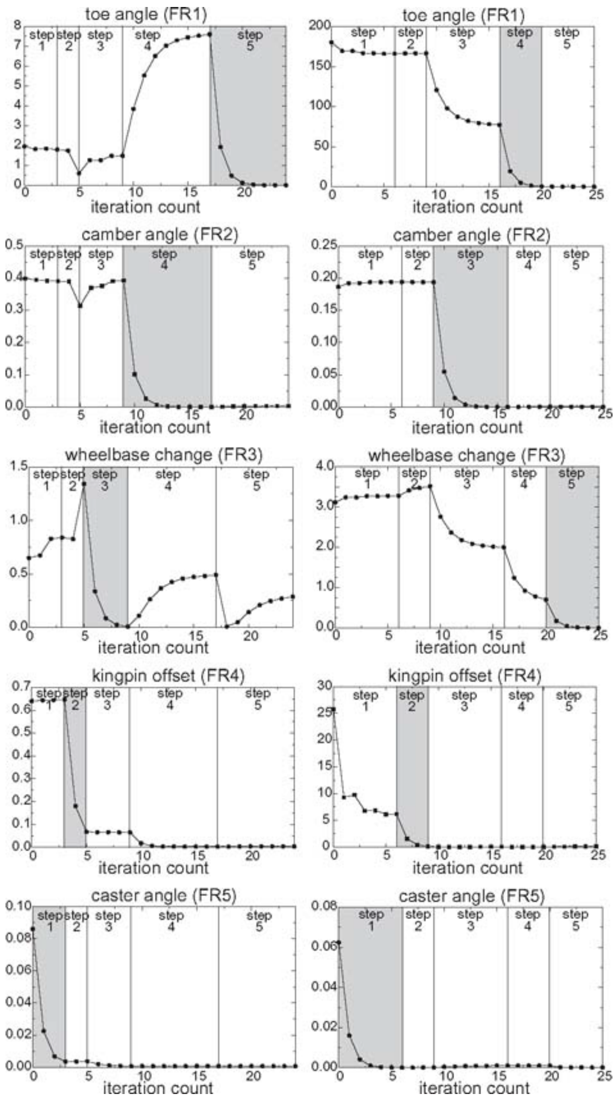
6 SEQUENTIAL DESIGN PROCEDURE

The design matrices for the suspension systems are almost triangular, which allows for sequential design. In a sequential design, a specific design order can satisfy all FRs. From the design equations, we observe that the design orders of the two cross-coupled designs are different from those of the two decoupled designs. That is to say, for two decoupled cases (multilink and design change) detailed design should start with the caster (FR_5), followed by the kingpin offset (FR_4), camber (FR_2), toe (FR_1) and wheelbase change (FR_3). On the other hand, for two cross-coupled cases (McPherson strut and double wishbone) the design order is FR_5, FR_4, FR_3, FR_2 and FR_1. In the design sequence, the order of wheelbase change (FR_3) is different, but the rest are identical.

To express a sequential design procedure mathematically, we need go back to Eq. (3). For example, if DP_j and DP_k are selected and grouped to form DP'_i, then the desired FR_i can be controlled by DP_j and DP_k; that is, DP_j is modified first with DP_k set to a constant value, and *vice versa*, iteratively. From Eq. (3), the increment of the hardpoint, which is selected as DP in each step, may be calculated as:

$$\Delta \mathrm{DP}_j = -\mathrm{A}_{ij}^{-1} \cdot \mathrm{FR}_i \qquad (8)$$

Figure 3 shows the traces of the FRs as the results of the sequential designs for two types of suspension systems. From the figure, desired FRs can be achieved by a sequential design. For the McPherson strut (see Figure 3(a)), FR_1 and FR_3 are coupled with FR_2, and FR_3 is coupled with FR_1. FR_1 can be minimized since FR_1 is behind FR_2, but FR_3 cannot be minimized because of design sequence. This can be expected from the design matrix of Eq. (4). For the multilink (see Figure 3(b)), five FRs are decoupled and can be minimized.



(a) McPherson strut (b) Multilink

Figure 3: Sequential design.

7 SUMMARY

This paper presents the sequential kinematic design of a suspension system based on Axiomatic Design (AD). AD is also used to compare the current design configurations of typical suspension systems and design modifications based on the AD is proposed.

Functional requirements (FRs) are defined from three independent suspension geometries (toe, camber and wheelbase) and two independent steering geometries (kingpin offset and caster). In addition, the corresponding design parameters (DPs) are selected using suspension hardpoints. Since the number of DPs is generally more than that of FRs, the method of grouping the DPs is presented. From the design equations, the multilink suspension results in a decoupled design, which is considered a good design compared with the other two types (McPherson strut and double wishbone). However, although the McPherson strut suspension is a cross-coupled design, a decoupled design can be achieved by replacing the coupled DPs with new DPs.

Suspension kinematic design has a specific order, because the design matrix is triangular. Design sequences can be classified into two groups: the cross-coupled case and the decoupled case. However, the sequences of the two groups have a common point: that is, the order of wheelbase is different, but the rest are identical.

Although this paper has presented the kinematic design of a suspension system, this methodology can be applied to kinematic design of general mechanical systems.

8 ACKNOWLEDGMENTS

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